
Motion of a Vortex Filament in an External Flow

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We consider a nonlinear model equation describing the motion of a vortex filament immersed in an incompressible and inviscid fluid. In the present problem setting, we also take into account the effect of external flow. We prove the unique solvability, locally in time, of an initial value problem posed on the one dimensional torus. The problem describes the motion of a closed vortex filament.

A vortex filament is a space curve on which the vorticity of the fluid is concentrated. Vortex filaments are used to model very thin vortex structures such as vortices that trail off airplane wings or propellers. In this talk, we prove the solvability of the following initial value problem which describes the motion of a closed vortex filament.

$$\begin{cases} \mathbf{x}_t = \frac{\mathbf{x}_s \times \mathbf{x}_{ss}}{|\mathbf{x}_s|^3} + \mathbf{F}(\mathbf{x}, t), & s \in \mathbf{T}, t > 0, \\ \mathbf{x}(s, 0) = \mathbf{x}_0(s), & s \in \mathbf{T}, \end{cases} \quad (1)$$

where $\mathbf{x}(s, t) = (x_1(s, t), x_2(s, t), x_3(s, t))$ is the position vector of the vortex filament parametrized by s at time t , the symbol \times is the exterior product in the three dimensional Euclidean space, $\mathbf{F}(\cdot, t)$ is a given external flow field, \mathbf{T} is the one dimensional torus \mathbf{R}/\mathbf{Z} , and subscripts are differentiations with the respective variables. Problem (1) describes the motion of a closed vortex filament under the influence of external flow. Such a setting can be seen as an idealization of the motion of a bubbling in water, where the thickness of the ring is taken to be zero and some environmental flow is also present. The main result we present in this talk is as follows.

Theorem 1 *For $T > 0$ and a natural number $m \geq 4$, if the initial filament \mathbf{x}_0 satisfies $\mathbf{x}_0 \in H^m(\mathbf{T})$ and $|\mathbf{x}_{0s}| \equiv 1$, and the external flow \mathbf{F} satisfies $\mathbf{F} \in C([0, T]; W^{m, \infty}(\mathbf{R}^3))$, then there exists $T_0 \in (0, T]$ such that a unique solution $\mathbf{x}(s, t)$ of (1) exists and satisfies*

$$\mathbf{x} \in C([0, T_0]; H^m(\mathbf{T})) \cap C^1([0, T_0]; H^{m-2}(\mathbf{T}))$$

The above theorem gives the time-local unique solvability of (1). We note that Nishiyama [1] proved the existence of a solution in $C([0, T]; H^2(\mathbf{T}))$ for any $T > 0$, and hence the above theorem extends his result to higher order Sobolev spaces and also ensures the uniqueness of the solution.

In this talk, the outline of the proof of Theorem 1 will be given, focusing only on the crucial parts.

References

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On Ground States of Spin-1 Bose-Einstein Condensates w/o external magnetic field

I-Liang CHERN

National Taiwan University & National Chiao Tung University

In this talk, I will first give a brief introduction to the spinor Bose-Einstein condensates (BECs). Then I will present two recent results, one is numerical, the other is analytical for spinor BECs w/o uniform external magnetic field.

In the numerical study of spinor BECs, a pseudo-arclength continuation method (PACM) was proposed for investigating the ground state patterns and phase diagrams of the spin-1 Bose-Einstein condensates under the influence of a homogeneous magnetic field.

Two types of phase transitions are found. The first type is a transition from a two-component (2C) state to a three-component (3C) state. The second type is a symmetry breaking in 3C state. After that, a phase separation of the spin component occurs. In the semi-classical regime, these two phase transition curves are gradually merged.

In the analytical study, the ground states of spin-1 BEC are characterized. First, we present the case when there is no external magnetic field. For ferromagnetic systems, we show the validity of the so-called single-mode approximation (SMA). For antiferromagnetic systems, there are two subcases. When the total magnetization $M \neq 0$, the corresponding ground states have vanishing zeroth ($m_F = 0$) components (so call 2C state), thus are reduced to two-component systems. When $M = 0$, the ground states are also reduced to the SMA, and there are one-parameter families of such ground states. Next, we study the case when an external magnetic field is applied. It is shown analytically that, for antiferromagnetic systems, there is a phase transition from 2C state to 3C state as the external magnetic field increases. The key idea in the proof is a redistribution of masses among different components, which reduces kinetic energy in all situations, and makes our proofs simple and unified.

The numerical part is a joint work with Jen-Hao Chen and Weichung Wang, whereas the analytical part is jointly with Liren Lin.

Darcy's law and diffusion of two-fluid Euler-Maxwell system with collisions

Renjun DUAN

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The talk is concerned with the large-time behavior of solutions to the Cauchy problem on the two-fluid Euler-Maxwell system with collisions when initial data are around a constant equilibrium state. The main goal is the rigorous justification of diffusion phenomena in fluid plasma at the linear level. Precisely, motivated by the classical Darcy's law for the nonconductive fluid, we first give a heuristic derivation of the asymptotic equations of the Euler-Maxwell

system in large time. It turns out that both the density and the magnetic field tend time-asymptotically to the diffusion equations with diffusive coefficients explicitly determined by given physical parameters. Then, in terms of the Fourier energy method, we analyze the linear dissipative structure of the system, which implies the almost exponential time-decay property of solutions over the high-frequency domain. The key part of the work is the spectral analysis of the linearized system, exactly capturing the diffusive feature of solutions over the low-frequency domain. Finally, under some conditions on initial data, we show the convergence of the densities and the magnetic field to the corresponding linear diffusion waves with the rate $(1+t)^{-5/4}$ in L^2 norm and also the convergence of the velocities and the electric field to the corresponding asymptotic profiles given in the sense of the generalized Darcy's law with the faster rate $(1+t)^{-7/4}$ in L^2 norm. Thus, the result can be also regarded as the mathematical proof of the Darcy's law in the context of collisional fluid plasma. This is a joint work with Qingqing Liu and Changjiang Zhu.

The Composite Waves in Hyperbolic Resonant Systems of Balance Laws

John HONG

National Central University

In this talk we consider some 2 by 2 and 3 by 3 hyperbolic resonant systems of balance laws arise in fluid dynamics, traffic flows and solid mechanics.

We say a system of conservation laws is resonant in domain ω if the eigenvalues to the Jacobian matrix of flux coincide in ω .

The existence, non-uniqueness and behavior of the entropy solutions to the Riemann problem is studied.

We introduce the new entropy waves for the Riemann problem of fully resonant systems. Those waves are the composition of Lax's elementary waves connecting to the vacuum state.

The stability of such composite waves is studied.

On the time periodic solutions and the asymptotic stabilities of GFDs and the related equations

Chun-Hsiung HSIA

National Taiwan University

In this lecture, we shall introduce time periodic solutions of differential equations arised from different occasions. We then turn our attention to the GFDs and study the asymptic stabilities of the solutions. In particular, two different approaches will be introduced to prove the existence of time periodic solutions of GFDs with time periodic focirings.

Sonic-Subsonic Limit of Approximate Solutions to Multidimensional Steady Euler Equations

Feimin HUANG

Chinese Academy of Sciences

A compactness framework is established for approximate solutions to sonic-subsonic flows governed by the steady full Euler equations for compressible fluids in arbitrary dimension. The existing compactness frameworks for the two-dimensional potential case do not directly apply for the steady full Euler equations in higher dimensions. In particular, the framework applies for both nonisentropic and rotational flows. One of our main observations is that the compactness can be achieved by using only natural weak estimates for the mass balance and the vorticity, along with the Bernoulli law and the entropy relation, through a more delicate analysis on the phase space. We then establish the compactness framework for approximate solutions to sonic-subsonic flows in any dimension. As direct applications, we establish several existence theorems for multidimensional sonic-subsonic for Euler flows through infinitely long nozzles. This talk is based on joint works with G.Q.Chen, T.Y.Wang and Y. Wang.

Geometric regularity criterion on the vorticity direction for the Navier-Stokes equations in the three-dimensional half space

Yasunori MAEKAWA

Tohoku University

In this talk we establish a geometric regularity criterion on the vorticity direction for the Navier-Stokes equations in the three-dimensional half space under the no-slip boundary condition. Our argument is based on a blow-up argument, which reduces the problem to a Liouville type problem for a backward global solution to the Navier-Stokes equations in the half plane with the no-slip boundary condition.

As is well known, the Liouville problem in an unbounded domain is deeply related with the behavior of the solution at spatial infinity, and the difficulty of our problem is that we can not assume any spatial decay for the solution in advance. We overcome this difficulty by analyzing the vorticity equations instead of the original Navier-Stokes equations. In particular, the boundary condition on the vorticity field plays a central role.

This is a joint work with Yoshikazu Giga (University of Tokyo) and Pen-Yuan Hsu (University of Tokyo).

Stationary wave to systems of viscous conservation laws in half line

Tohru NAKAMURA

Kyushu University

We consider the large-time behavior of solutions to the symmetric hyperbolic-parabolic system in the half line. We firstly prove the existence of the stationary solution by assuming that a boundary strength is sufficiently small. We next prove that the stationary solution is time

asymptotically stable under a smallness assumption on the initial perturbation. The key to proof is to derive the uniform a priori estimates by using the energy method in half space developed by Matsumura and Nishida as well as the stability condition of Shizuta–Kawashima type. The present talk is based on the joint research with Professor Shinya Nishibata at Tokyo Institute of Technology.

Asymptotic stability of stationary solutions to the Euler-Poisson equation arising in plasma physics

Shinya NISHIBATA

Tokyo Institute of Technology

The main concern of the present talk is mathematical analysis on a boundary layer around a surface of a material with which plasma contacts. The layer, called a sheath in plasma physics, has a larger density of positive ions than that of electrons. The Bohm criterion for formation of the sheath requires that ion velocity should be hyper-sonic. This physical phenomenon is studied in the Euler-Poisson equations describing behavior of ionized gas. We show that the sheath is regarded as a planar stationary solution in multi-dimensional half space. Precisely, under the Bohm sheath criterion, we prove the existence of the stationary solution and its time asymptotic stability. Moreover we obtain a convergence rate of the solution towards the stationary solution. If we have enough time, I will show results by numerical experiments.

Analysis of Nematic Liquid Crystals with Half Singularities

Jinhae PARK

Chungnam National University

We investigate the structure of nematic liquid crystal thin films described by the Landau-de Gennes tensor-valued order parameter model with Dirichlet boundary conditions on the sides of nonzero degree. We prove that as the elasticity constant goes to zero in the energy, a limiting uniaxial nematic texture forms with a finite number of defects, all of degree 1/2 or all of degree -1/2, corresponding to vertical disclination lines at those locations.

One-Dimensional Non-isentropic Euler Equations with Periodic Initial Data

Peng QU

The Chinese University of Hong Kong

In this talk, the non-isentropic Euler system with periodic initial data in \mathbb{R}^1 is discussed by analyzing wave interactions in a framework of specially chosen Riemann invariants, generalizing Glimm functional and applying the method of approximate conservation laws and approximate characteristics. An $\mathcal{O}(\varepsilon^{-2})$ lower bound is established for the life span of the entropy solutions with initial data that possess ε variation in each period.

**Motion of the inviscid gas through a nozzle
-Existence of a time global solution and invariant regions-**

Naoki TSUGE
Gifu University

We are concerned with the motion of the gas through nozzles. The phenomena are governed by the compressible Euler equations.

First, we study the steady flow to review how the nozzle flow is applied in physics and engineering.

Next, we consider the insteady flow, in particular, the existence of a time global solution for the present problem. For this problem, if initial data are away from the sonic state, the existence theorem has been obtained by Liu [2]. On the other hand, the transonic flow is essential for engineering and physics. Therefore, our goal is to prove the existence of solutions for the transonic flow. In this talk, we introduce two main theorems for the Laval nozzle [3] and the general nozzle [4]. Then, the most difficult point is to obtain bounded estimates of solutions. Invariant regions have been the major method to obtain the estimates until now. However, we cannot apply the standard invariant region theory [1] to the present problem. To solve this problem, we employ new invariant regions, which depend on a space variable.

References

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- [2] Liu, T.-P., Quasilinear hyperbolic systems. *Commun. Math. Phys.* **68**, 141–172 (1979)
- [3] Tsuge, N., Existence of global solutions for unsteady isentropic gas flow in a Laval nozzle. *Arch. Ration. Mech. Anal.* **205**, 151–193 (2012)
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**Decay structure of the regularity-loss type and the asymptotic stability for the
Euler-Maxwell system**

Yoshihiro UEDA
Kobe University

In this talk, we consider the Cauchy problem of the Euler-Maxwell system. The Euler-Maxwell system describes the dynamics of compressible electrons in plasma physics under the interaction of the magnetic and electric fields via the Lorentz force. Our purpose is to study the large-time behavior of solutions to the initial value problem for the Euler-Maxwell system in whole space. This system verifies the decay property of the regularity-loss type. Under smallness condition on the initial perturbation, we show that the solution to the problem exists globally in time and converges to the equilibrium state and stationary solution. Moreover we derive the corresponding convergence rate of the solutions. The key to the proof of our main theorems are to derive a priori estimates of solutions by using the energy method.

Smooth Transonic Flows in De Laval Nozzles

Chunpeng WANG

Jilin University & The Chinese University of Hong Kong

This talk concerns smooth transonic flows of Meyer type in de Laval nozzles, which are governed by an equation of mixed type with degeneracy and singularity at the sonic state. First we study the properties of sonic curves. For any C2 transonic flow of Meyer type, the set of exceptional points is shown to be a closed line segment (may be empty or only one point). Furthermore, it is proved that a flow with nonexceptional points is unstable for a C1 small perturbation in the shape of the nozzle. Then we seek smooth transonic flows of Meyer type which satisfy physical boundary conditions and whose sonic points are exceptional. For such a flow, its sonic curve must be located at the throat of the nozzle and it is strongly singular in the sense that the sonic curve is a characteristic degenerate boundary in the subsonic-sonic region, while in the sonic-supersonic region all characteristics from sonic points coincide, which are the sonic curve and never approach the supersonic region. It is proved that there exists uniquely such a smooth transonic flow near the throat of the nozzle, whose acceleration is Lipschitz continuous, if the wall of the nozzle is sufficiently flat. The global extension of this local smooth transonic flow is also studied. The works are jointed with Professor Zhouping Xin.

Zero surface tension limit of viscous surface waves

Yanjin WANG

The Chinese University of Hong Kong

We consider the free boundary problem for a layer of viscous, incompressible fluid in a uniform gravitational field, lying above a rigid bottom and below the atmosphere. We establish the zero surface tension limit of the problem, both local and global in time. This is a joint work with Prof. Zhong Tan.

Spectrum Analysis of some kinetic equations

Tong YANG

The City University of Hong Kong

In this talk, we will first present a general framework for analyzing the spectrum structure of kinetic equations and apply it to the Boltzmann equation without angular cutoff and the Landau equation. And then we will present some results on the spectrum analysis for the Vlasov-Poisson-Boltzmann system and the Vlasov-Maxwell-Boltzmann system. These are some joint works with Hongjun Yu, Hailiang Li and Mingying Zhong.

Partial Regularity for the fractional Navier-Stokes equation

Yong YU

The Chinese University of Hong Kong

We are going to talk about the partial regularity for the fractional Navier-Stokes equation. The results generalize the famous result of Caffarelli-Kohn-Nirenberg and F.H.Lin for the standard Navier-Stokes equation in 3D.

The Vlasov-Poisson-Boltzmann System near Maxwellians in the Whole Space

Huijiang ZHAO

Wuhan University

This talk is concerned with some recent progress on the construction of global smooth solutions to the one-species Vlasov-Poisson-Boltzmann system near Maxwellians in the whole space for the whole range of cutoff intermolecular interactions.

Regularity criteria for the 3D Navier-Stokes and MHD equations via partial components

Yong ZHOU

Zhejiang Normal University

In this talk, I will talk about some recent developments on regularity criteria for the 3D Navier-Stokes and MHD equations via partial components. Difficulties and open questions will be also discussed.